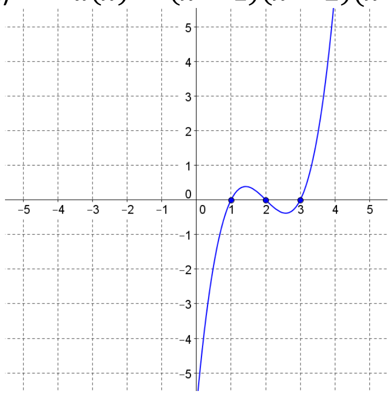
SM3 2.5 Graphing Polynomials

Sketch the polynomial with accurate roots and end behavior. Discuss the end behavior of the polynomial using limit notation.

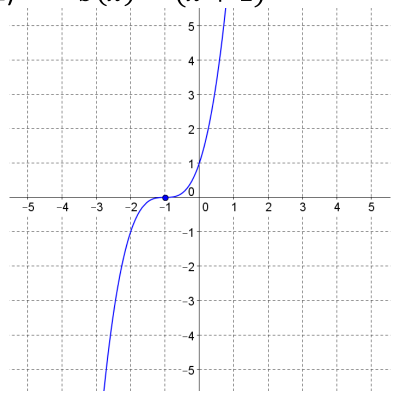
1) $a(x) = (x-1)(x-2)(x-3)$



$\lim_{x\to -\infty} a(x) = -\infty$

$\lim_{x\to \infty} a(x) = \infty$

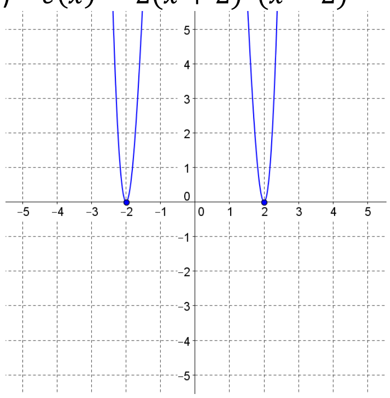
2) $b(x) = (x+1)^3$



$\lim_{x\to -\infty} b(x) = -\infty$

$\lim_{x\to \infty} b(x) = \infty$

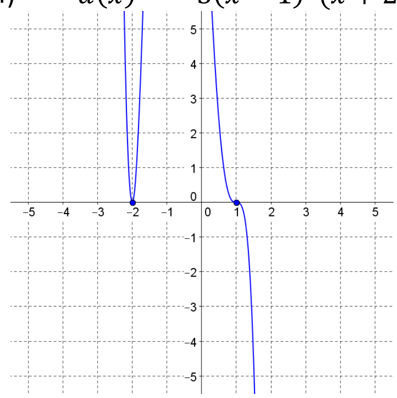
3) $c(x) = 2(x+2)^2(x-2)^2$



$\lim_{x\to -\infty} c(x) = \infty$

$\lim_{x\to \infty} c(x) = \infty$

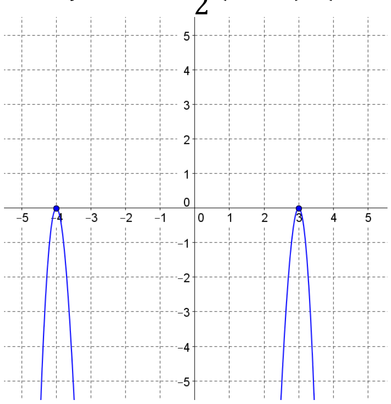
4) $d(x) = -3(x-1)^3(x+2)^2$



$\lim_{x\to -\infty} d(x) = \infty$

$\lim_{x\to \infty} d(x) = -\infty$

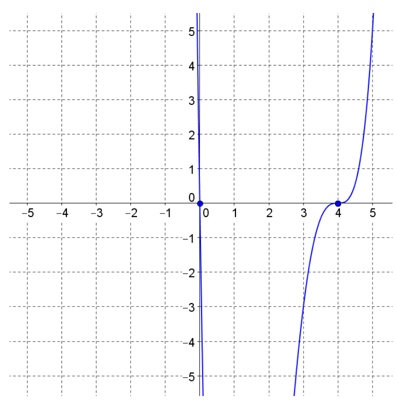
5) $f(x) = -\frac{1}{2}(x+4)^2(x-3)^2$



$\lim_{x\to -\infty} f(x) = -\infty$

$\lim_{x\to \infty} f(x) = -\infty$

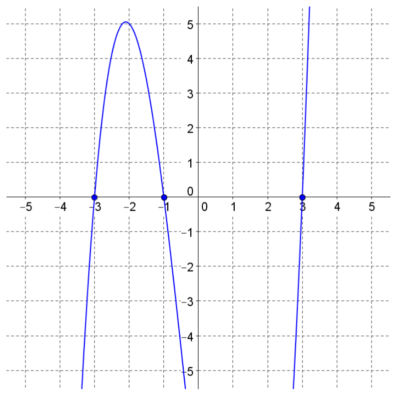
6) $g(x) = x(x-4)^3$



$\lim_{x\to -\infty} g(x) = \infty$

$\lim_{x\to \infty} g(x) = \infty$

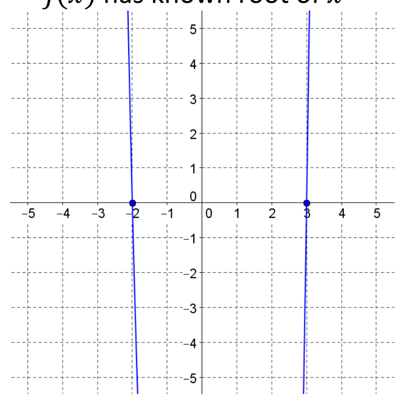
7) $h(x) = x^3 + x^2 - 9x - 9$



$\lim_{x \to -\infty} h(x) = -\infty$

$\lim_{x \to \infty} h(x) = \infty$

8) $j(x) = x^4 - x^3 - 2x^2 - 4x - 24$
$j(x)$ has known root of $x = -2i$



$\lim_{x \to -\infty} j(x) = \infty$

$\lim_{x \to \infty} j(x) = \infty$

Problems: State the least degree polynomial, in descending order, that has the given roots and satisfies the condition:

9) $x = \{-2, 5\}; f(1) = -36$

$$\begin{aligned} a(1+2)(1-5) &= -36 \\ a(3)(-4) &= -36 \\ -12a &= -36 \\ a &= 3 \end{aligned}$$

$f(x) = 3(x+2)(x-5)$
$f(x) = 3(x^2 - 3x - 10)$

$f(x) = 3x^2 - 9x - 30$

10) $x = \left\{-3, \frac{1}{2}\right\}; f(1) = 8$

$$\begin{aligned} a(1+3)(2(1)-1) &= 8 \\ a(4)(1) &= 8 \\ 4a &= 8 \\ a &= 2 \end{aligned}$$

$f(x) = 2(x+3)(2x-1)$
$f(x) = 2(2x^2 + 5x - 3)$

$f(x) = 4x^2 + 10x - 6$

11) $x = \{-5i, 5i\}; f(1) = -52$

$$\begin{aligned} a(1+5i)(1-5i) &= -52 \\ a(1+25) &= -52 \\ 26a &= -52 \\ a &= -2 \end{aligned}$$

$f(x) = -2(x+5i)(x-5i)$
$f(x) = -2(x^2 + 25)$

$f(x) = -2x^2 - 50$

12) $x = \left\{-1, -\frac{3}{4}\right\}; f(2) = \frac{33}{2}$

$$\begin{aligned} a(2+1)(4(2)+3) &= \frac{33}{2} \\ a(3)(11) &= \frac{33}{2} \\ 33a &= \frac{33}{2} \\ a &= \frac{1}{2} \end{aligned}$$

$f(x) = \frac{1}{2}(x+1)(4x+3)$
$f(x) = \frac{1}{2}(4x^2 + 7x + 3)$
$f(x) = 2x^2 + \frac{7}{2}x + \frac{3}{2}$

13) $x = \left\{\frac{2}{3}, \frac{3}{2}\right\}; f(-1) = 50$

$$\begin{aligned} a(3(-1)-2)(2(-1)-3) &= 50 \\ a(-5)(-5) &= 50 \\ 25a &= 50 \\ a &= 2 \end{aligned}$$

$f(x) = 2(3x-2)(2x-3)$
$f(x) = 2(6x^2 - 13x + 6)$

$f(x) = 12x^2 - 26x + 12$

14) $x = \{4 - i, 4 + i\}; f(1) = 10$

$$\begin{aligned} a(1-4+i)(1-4-i) &= 10 \\ a(-3+i)(-3-i) &= 10 \\ a(9+1) &= 10 \\ 10a &= 10 \\ a &= 1 \end{aligned}$$

$f(x) = (x-4+i)(x-4-i)$
$f(x) = (x^2 - 8x + 17)$

$f(x) = x^2 - 8x + 17$

15) $x = \{-3, 0, 3\}; f(2) = -5$

$$\begin{aligned} a(2+3)(2+0)(2-3) &= -5 \\ a(5)(2)(-1) &= -5 \\ -10a &= -5 \\ a &= \frac{1}{2} \end{aligned}$$

$$f(x) = \frac{1}{2}(x+3)(x+0)(x-3)$$

$$f(x) = \frac{1}{2}x(x^2-9)$$

$$f(x) = \frac{1}{2}(x^3-9x)$$

$$f(x) = \frac{1}{2}x^3 - \frac{9}{2}x$$

16) $x = \{-2$ w/ m. of 2, $2\}$; $f(1) = -27$

$$\begin{aligned} a(1+2)(1+2)(1-2) &= -27 \\ a(3)(3)(-1) &= -27 \\ -9a &= -27 \\ a &= 3 \end{aligned}$$

$$f(x) = 3(x+2)(x+2)(x-2)$$
$$f(x) = 3(x+2)(x^2-4)$$
$$f(x) = 3(x^3+2x^2-4x-8)$$

$$f(x) = 3x^3 + 6x^2 - 12x - 24$$

17) $x = \{-6, -4, 5\}; f(1) = -70$

$$\begin{aligned} a(1+6)(1+4)(1-5) &= -70 \\ a(7)(5)(-4) &= -70 \\ -140a &= -70 \\ a &= \frac{1}{2} \end{aligned}$$

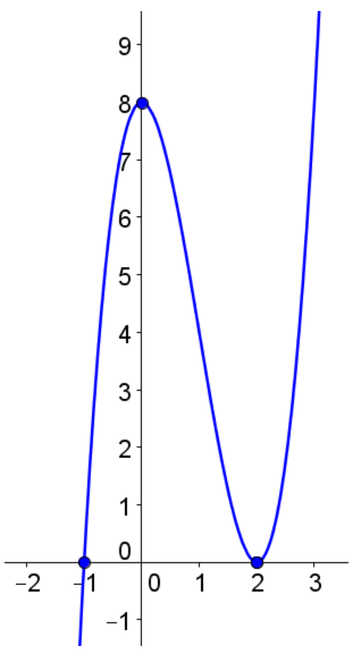
$$f(x) = \frac{1}{2}(x+6)(x+4)(x-5)$$

$$f(x) = \frac{1}{2}(x+6)(x^2-x-20)$$

$$f(x) = \frac{1}{2}(x^3+5x^2-26x-120)$$

$$f(x) = \frac{1}{2}x^3 + \frac{5}{2}x^2 - 13x - 60$$

Cumulative Problem: Build and graph a **cubic** polynomial to meet the following specifications:



Roots: $x = \{-1, 2\}$

Increasing interval: $(-\infty, 0) \cup (2, \infty)$
Decreasing interval: $(0, 2)$

Condition: $f(0) = 8$

End Behavior:
$\lim_{x \to -\infty} f(x) = -\infty$ $\lim_{x \to \infty} f(x) = \infty$

$$f(x) = 2(x+1)(x-2)^2$$
$$f(x) = 2x^3 - 6x^2 + 8$$